Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

■ : explanation
∵ is 'because'
∴ is 'therefore'

Pearson Edexcel International Advanced Level

Time 1 hour 30 minutes

Paper reference **WMA11/01**

Mathematics

January 2023

International Advanced Subsidiary/Advanced Level

Pure Mathematics P1

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 11 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

Pearson

1. A curve C has equation

$$y = 2 + 10x^{\frac{1}{2}} - 2x^{\frac{3}{2}} \qquad x > 0$$

(a) Find $\frac{dy}{dx}$ giving your answer in simplest form.

(3)

(b) Hence find the exact value of the gradient of the tangent to C at the point where $x = 2$ giving your answer in simplest form.

(Solutions relying on calculator technology are not acceptable.)

(2)

$\because x^0 = 1$

a) $y = 2x^0 + 10x^{1/2} - 2x^{3/2}$

$dy/dx = 0(2x^{0-1}) + \frac{1}{2}(10x^{\frac{1}{2}-1}) + \frac{3}{2}(-2x^{\frac{3}{2}-1}) = 0 + 5x^{-1/2} - 3x^{\frac{1}{2}}$

$dy/dx = 5x^{-\frac{1}{2}} - 3x^{\frac{1}{2}}$

b) tangent means gradient of tangent is same as gradient of equation

to find gradient of tangent, substitute $x = 2$ into dy/dx (the gradient function)

$dy/dx = 5(2)^{-1/2} - 3(2)^{1/2} = \frac{5}{\sqrt{2}} - 3\sqrt{2}$

$\times\sqrt{2}$

$= \frac{5}{\sqrt{2}} - \frac{3\sqrt{2}}{1} = \frac{5}{\sqrt{2}} - \frac{(3\times2)}{\sqrt{2}} = \frac{5}{\sqrt{2}} - \frac{6}{\sqrt{2}} = \frac{-6}{\sqrt{2}} = \frac{-1}{\sqrt{2}}$

$\times\sqrt{2}$

RATIONALISE : $= \frac{-1}{\sqrt{2}} \frac{\times\sqrt{2}}{\times\sqrt{2}} = \frac{-1\sqrt{2}}{2} = -\frac{1}{2}\sqrt{2}$

$\therefore$ gradient is $-\frac{1}{2}\sqrt{2}$

Question 1 continued

(Total for Question 1 is 5 marks)

2. The points P, Q and R have coordinates (–3, 7), (9, 11) and (12, 2) respectively.

(a) Prove that angle $PQR = 90°$

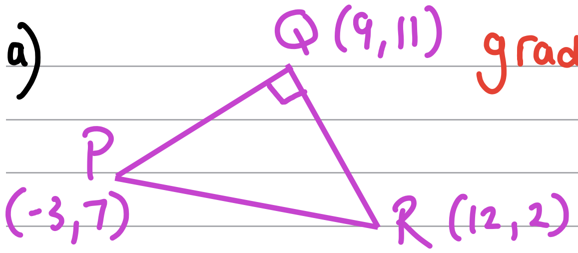
(3)

Given that the point S is such that $PQRS$ forms a rectangle,

(b) find the coordinates of S.

(2)

a)



gradient formula : $m = \frac{y_1 - y_2}{x_1 - x_2}$

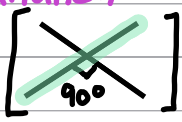
① find gradient of PQ.

$$M_{PQ} = \frac{7-11}{-3-9} = \frac{-4}{-12} = \frac{-4(1)}{-4(3)} = \frac{1}{3}$$

② find gradient of QR.

$$M_{QR} = \frac{11-2}{9-12} = \frac{9}{-3} = -3$$

Explanation: <PQR is 90°. A normal is perpendicular (90°) to a line



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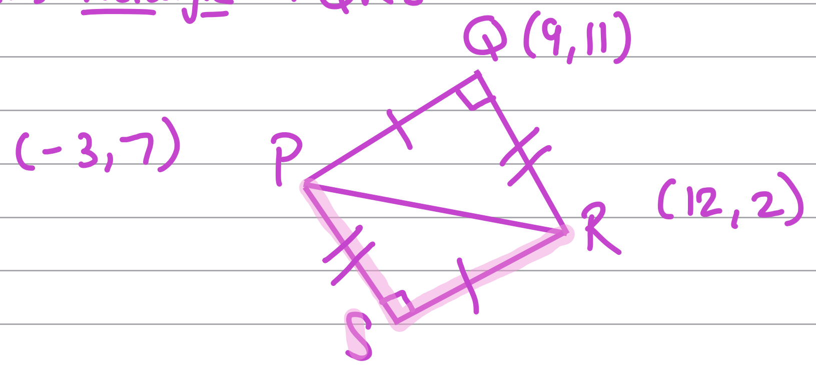
Perpendicular gradient rule states that : $M_{normal} \times M_{line} = -1$

$$M_{PQ} \times M_{QR} = \frac{1}{3} \times -3 = -1$$

meaning that lines PQ and QR are perpendicular as gradient PQ = $\frac{1}{3}$ & gradient QR = -3 ∴ angle PQR = 90°

Question 2 continued

b) S forms rectangle PQRS:



1 find equation of line RS

gradient PQ = $\frac{1}{3}$ which will be the same as gradient RS ∵ Side RS = PQ

∴ equation RS:

[point R(12,2), gradient $\frac{1}{3}$]

line passing through (a, b) & gradient m

equation: $(y-b) = m(x-a)$

a = 12
b = 2
m = 1/3

$(y-2) = \frac{1}{3}(x-12)$

$y - 2 = \frac{1}{3}x - 4$

2 find equation of line PS

gradient QR = −3 which will be the same as gradient PS ∵ Side PS = QR

∴ equation PS:

[point (−3,7), gradient −3]

line passing through (a, b) & gradient m

equation: $(y-b) = m(x-a)$

a = −3
b = 7
m = −3

$(y-7) = -3(x-(-3))$

$y - 7 = -3x - 9$

(Total for Question 2 is 5 marks)

3 Solve simultaneous equations:

RS: $y - 2 = \frac{1}{3}x - 4$

$\Downarrow$

$y = \frac{1}{3}x - 2$

(simplifying)

PS: $y - 7 = -3x - 9$

$\Downarrow$

$y = -3x - 2$

$\Rightarrow$ RS: $y = \frac{1}{3}x - 2$

PS: $y = -3x - 2$ $-$

Solving for x.

$0 = \frac{10}{3}x + 0$

$0 = \frac{10}{3}x$

$\therefore x = 0$

Substitute x-value into equation RS or PS to find y-coordinate.

PS: $y = -3x - 2$

$y = -3(0) - 2$

$\therefore y = -2$

$\therefore S(0, -2)$

3. Find

$$\int \frac{4x^5+3}{2x^2}\,dx$$

giving your answer in simplest form.

(5)

Integration Method.

① Write in easier form for integration.

$$\frac{4x^5+3}{2x^2} = \frac{4x^5}{2x^2} + \frac{3}{2x^2} = \frac{2x^5}{x^2} + \left(\frac{3}{2}\times\frac{1}{x^2}\right)$$

$$= \frac{2x^5}{x^2} + \frac{3}{2}\left(\frac{1}{x^2}\right) = 2x^{5-2} + \frac{3}{2}x^{-2}$$

① indicies rule $\frac{a^b}{a^c} = a^{b-c}$ ② indicies rule $\frac{a}{x^b} = ax^{-b}$

$$= 2x^3 + \frac{3}{2}x^{-2}$$

② Integration.

$$\int 2x^3 + \frac{3}{2}x^{-2}\,dx = \left[\left(\frac{2}{3+1}x^{3+1}\right) + \left(\frac{3/2}{-2+1}x^{-2+1}\right)\right] = \frac{2}{4}x^4 + \frac{3/2}{-1}x^{-1}$$

$$= \frac{1}{2}x^4 - \frac{3}{2}x^{-1} + C$$

↳ DON'T FORGET or will lose a mark

$$\therefore \boxed{\frac{1}{2}x^4 - \frac{3}{2}x^{-1} + C}$$

P 7 2 0 6 6 A 0 6 3 2

Question 3 continued

(Total for Question 3 is 5 marks)

4. Given that the equation

$$kx^2 + 6kx + 5 = 0 \qquad \text{where } k \text{ is a non zero constant}$$

has no real roots, find the range of possible values for k.

(4)

Use the discriminant formula $b^2 - 4ac < 0$ for no real roots

no real roots means that when the equation of the graph is $ax^2 + bx + c = 0$
discriminant is $b^2 - 4ac < 0$

$$kx^2 + 6kx + 5 = 0$$

$a = k$
$b = 6k$
$c = 5$

$$b^2 - 4ac < 0$$
$$(6k)^2 - 4(k)(5) < 0$$
$$36k^2 - 20k < 0$$

Solve for k when $36k^2 - 20k < 0$

1) factorise : $4k(9k - 5) = 0$

2) Solve : $4k = 0$ $\therefore k = 0$ $9k - 5 = 0 \Rightarrow 9k = 5$ $\therefore k = 5/9$

$\therefore$ Critical points $k = 0, 5/9$

$\because b^2 - 4ac < 0$ NOT $b^2 - 4ac \leq 0$

Consider k is one less than 0, $k = -1$: $36(-1)^2 - 20(-1) = 56$
$56 > 0$ $\therefore 0 < k$

Consider k is one more than $5/9$, $k = 14/9$: $36(14/9)^{-1} - 20(14/9) = 56$
$56 > 0$ $\therefore k < 5/9$

$$\therefore \quad 0 < k < 5/9$$

P 7 2 0 6 6 A 0 8 3 2

Question 4 continued

(Total for Question 4 is 4 marks)

5. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

(a) By substituting $p = 3^x$, show that the equation

$$3 \times 9^x + 3^{x+2} = 1 + 3^{x-1}$$

can be rewritten in the form

$$9p^2 + 26p - 3 = 0$$

(3)

(b) Hence solve

$$3 \times 9^x + 3^{x+2} = 1 + 3^{x-1}$$

(3)

a) Isolate the 3^x to be able to substitute in $p = 3^x$

$3 \times 9^x + 3^{x+2} = 1 + 3^{x-1}$

$3 \times 9^x + (3^x \times 3^2)$ ① $= 1 + (3^x \div 3^1)$ ②

① indicies rule: $a^{b+c} = a^b \times a^c$ ② indicies rule $ax^{-b} = \frac{a}{x^b}$

$3 \times 9^x + (3^x \times 3^2) = 1 + (3^x \div 3^1)$

$3 \times (3^2)^x + (3^x \times 3^2) = 1 + (3^x \div 3)$

$3 \times (3^x)^2$ ③ $+ 3^x \times 3^2 = 1 + (3^x \div 3)$

③ indicies rule: $a^{bc} = (a^b)^c = (a^c)^b$

$\therefore \quad 3(3^x)^2 + (3^x \times 3^2) = 1 + (3^x \div 3)$

Substitute $p = 3^x$

$3(3^x)^2 + (3^x \times 3^2) = 1 + (3^x \div 3)$

$3(p)^2 + (p \times 9) = 1 + (p \div 3)$

$\times 3 \quad 3p^2 + 9p = 1 + p/3 \quad \times 3$

$-p \quad 9p^2 + 27p = 3 + p \quad -p$

$-3 \quad 9p^2 + 26p = 3 \quad -3$

$9p^2 + 26p - 3 = 0$

$\therefore \; 9p^2 + 26p - 3 = 0$

DO NOT WRITE IN THIS AREA

P 7 2 0 6 6 A 0 1 0 3 2

Question 5 continued

b) We will first solve using equation from part (a)

$9p^2+26p-3=0$

factorise : $(9p-1)(p+3)=0$

Solve : 1) $9p-1=0$ 2) $p+3=0$

$9p=1$ $\therefore p=-3$

$\therefore p=1/9$

Substitute $p=3^x$

$p=1/9 \implies 3^x=1/9$

Method ①: 1) $1/9=9^{-1}$ ∵ indicies rule $\frac{1}{x^b}=1x^{-b}$

2) $9^{-1}=(3^2)^{-1}=3^{-2}$ ∵ indicies rule $a^{bc}=(a^b)^c$

3) $3^x=3^{-2}$ $\therefore x=-2$

Method ②: if you have already done Pure 2 logarithms

1) when $a^x=n$ $\log_a n=x$

2) $3^x=1/9$ $\log_3 \frac{1}{9}=x$ type into calculator

3) $\log_3 \frac{1}{9}=-2$ $\therefore x=-2$

$p=-3 \implies 3^x\neq-3$

x is undefined. Negative number cannot be expressed as a power of a positive base (in this Question positive base is 3)

using logarithms:

from Pure 2 where: $a^x=n$

$\log_a n=x$

n CANNOT be a negative number

$\therefore x=-2$

(Total for Question 5 is 6 marks)

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6.

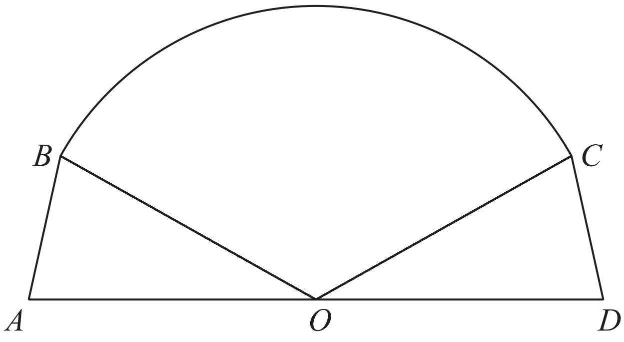


Diagram **NOT** accurately drawn

Figure 1

Figure 1 shows the plan view for the design of a stage.

The design consists of a sector OBC of a circle, with centre O, joined to two congruent triangles OAB and ODC.

Given that

units

- angle BOC = 2.4 radians
- area of sector BOC = $40\,\text{m}^2$
- AOD is a straight line of length 12.5 m

(a) find the radius of the sector, giving your answer, in m, to 2 decimal places, **(2)**

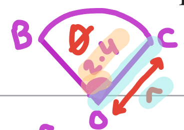
(b) find the size of angle AOB, in radians, to 2 decimal places. **(1)**

Hence find

(c) the total area of the stage, giving your answer, in m^2, to one decimal place, **(3)**

(d) the total perimeter of the stage, giving your answer, in m, to one decimal place. **(4)**

a) formula for Area of sector : $A = \frac{1}{2}r^2\theta$



$A = 40\text{m}^2$
$\theta = 2.4$ rad → Substitute into formula & solve for r

$$40 = \frac{1}{2}r^2(2.4)$$

$$40 = 1.2r^2$$

÷1.2 $\quad \frac{100}{3} = r^2 \quad$ ÷1.2

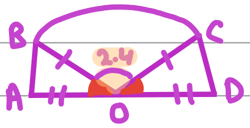
Square root $\quad \sqrt{\frac{100}{3}} = \sqrt{r^2} \quad$ square root

$$\frac{10\sqrt{3}}{3} = r$$

$r = \frac{10\sqrt{3}}{3} = 5.7735026\ldots$

$\therefore r = 5.77\text{ m}$ (2dp)

Question 6 continued

b) 

Angles on a straight line add up to π radians (180°).

$\angle BOA = \angle COD$

$\therefore 2.4 + 2(\angle BOA) = \pi$

$\angle BOA = \dfrac{\pi - 2.4}{2} = 0.370796\ldots \approx 0.37$

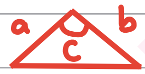
$\therefore \angle BOA = 0.37 \text{ rad}$ (2dp)

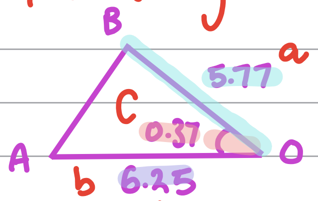
c) Total Area $= A_{BOC} + A_{AOB} + A_{COD} = A_{BOC} + 2(A_{AOB})$

$\because A_{AOB} = A_{COD}$

To find Area of AOB / COD :

Area of a triangle : $A = \frac{1}{2}ab\sin C$





$A = \frac{1}{2}(5.77)(6.25)\sin 0.371 = 18.03125\sin 0.371$

$= 6.537186\ldots \approx 6.54$

Obtained by dividing line AOD (12.5m) by 2.

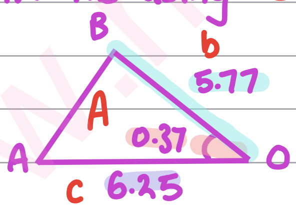
Total Area $= 40 + 2(6.54) = 53.08 \text{ m}^2$

$\therefore$ Total Area $= 53.1 \text{ m}^2$ (1dp)

d) Perimeter $= AB + BC + CD + AOD = 2AB + BC + AOD$

$\because AB = CD$

① find length AB using Cosine rule



Pure Mathematics P1

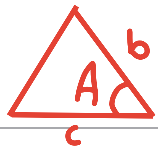
Mensuration

Surface area of sphere $= 4\pi r^2$

Area of curved surface of cone $= \pi r \times$ slant height

Cosine rule

$a^2 = b^2 + c^2 - 2bc\cos A$



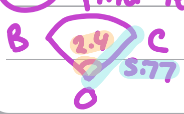
$AB^2 = 5.77^2 + 6.25^2 - 2(5.77)(6.25)\cos 0.37$

$AB^2 = 72.3554 - 72.125\cos 0.37$

$AB^2 = 5.11129\ldots$

$\therefore AB = 2.2608\ldots \approx 2.26$

② find length of arc BC using $S = r\theta$



$S = 5.77 \times 2.4 = 13.848 \approx 13.85 \text{ m}$

Question 6 continued

(3) Total perimeter $= 2AB + BC + AOD$

$= 2(2.26) + 13.85 + 12.5$

$= 30.87$

$\therefore$ Total Perimeter $= 30.9\text{ m}$ (1dp)

P 7 2 0 6 6 A 0 1 4 3 2

Question 6 continued

(Total for Question 6 is 10 marks)

7. (a) On Diagram 1, sketch a graph of the curve C with equation

$$y = \frac{6}{x} \qquad x \neq 0$$

(2)

The curve C is transformed onto the curve with equation $y = \dfrac{6}{x-2} \qquad x \neq 2$

(b) Fully describe this transformation.

(2)

The curve with equation

$$y = \frac{6}{x-2} \qquad x \neq 2$$

and the line with equation

$$y = kx + 7 \qquad \text{where } k \text{ is a constant}$$

intersect at exactly two points, P and Q.

Given that the x coordinate of point P is -4

(c) find the value of k,

(2)

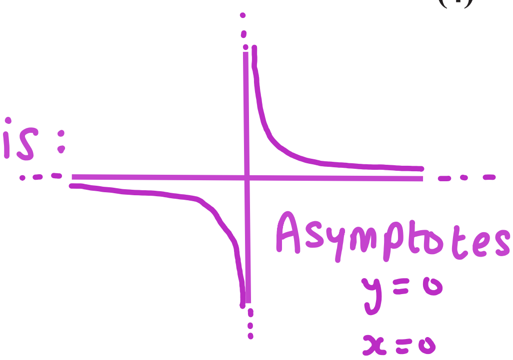
(d) find, using algebra, the coordinates of point Q.

(*Solutions relying entirely on calculator technology are not acceptable.*)

(4)

a) Working out:

$y = \frac{6}{x}$ graph of $y = \frac{1}{x}$ is:



Asymptotes:
$y = 0$
$x = 0$

if we let $y = f(x) = \frac{1}{x}$

$y = \frac{6}{x} = 6\left(\frac{1}{x}\right)$ is $y = 6f(x)$ so $y = \frac{6}{x}$ is the vertical stretch of $y = \frac{1}{x}$ by factor 6. Asymptotes are unchanged.

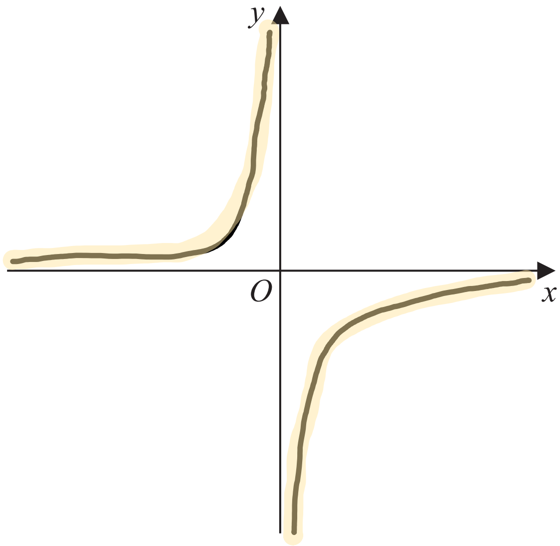
$\therefore$ $y = 0 \times 6 = 0$

$x = 0$ (x-coordinates are unchanged)

P 7 2 0 6 6 A 0 1 6 3 2

Mark Scheme :

Question 7 continued



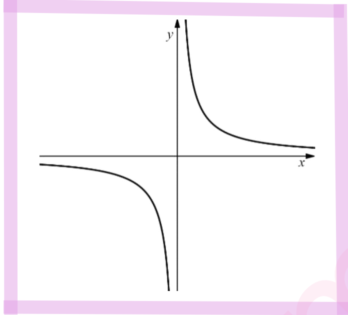


Diagram 1

b) $y = \frac{6}{x-2}$

Consider letting $y = \frac{6}{x} = f(x)$, so $y = \frac{6}{x-2}$ is $y = f(x-2)$ so this is translation $\binom{2}{0}$ meaning graph is translated 2 units to the right.

$\therefore$ translation is 2 units to the right (parallel to the x-axis)

c) $y = \frac{6}{x-2}$ & $y = kx + 7$ intersect $\therefore$ equate them together.

$$\frac{6}{x-2} = kx + 7$$

Substitute P x-coordinate $x = -4$.

$$\frac{6}{-4-2} = k(-4) + 7$$

Solve for K.

$$6/-6 = -4k + 7$$

$$-1 = -4k + 7$$

$$-7 \quad -8 = -4k \quad -7$$

$$\div -4 \quad 2 = k \quad \div -4$$

$$\therefore k = 2$$

Turn over for a copy of Diagram 1 if you need to redraw your graph.

Question 7 continued

d) equate $y = \frac{6}{x-2}$ & $y = kx + 7$ together

from part (c)

$$\frac{6}{x-2} = 2x + 7$$

Form quadratic equation.

$$\frac{6}{x-2} = 2x + 7$$

$\times (x-2)$ $$6 = (2x+7)(x-2)$$ $\times (x-2)$

$$6 = 2x^2 - 4x + 7x - 14$$

-6 $$0 = 2x^2 + 3x - 20$$ -6

Solve for x with $2x^2 + 3x - 20 = 0$

Factorise $(2x-5)(x+4) = 0$

Solve $2x - 5 = 0$ $\quad\quad$ $x + 4 = 0$

$2x = 5$ $\quad\quad$ $\therefore x = -4$ this is x-coordinate of Point P given in question

$\therefore x = 5/2$

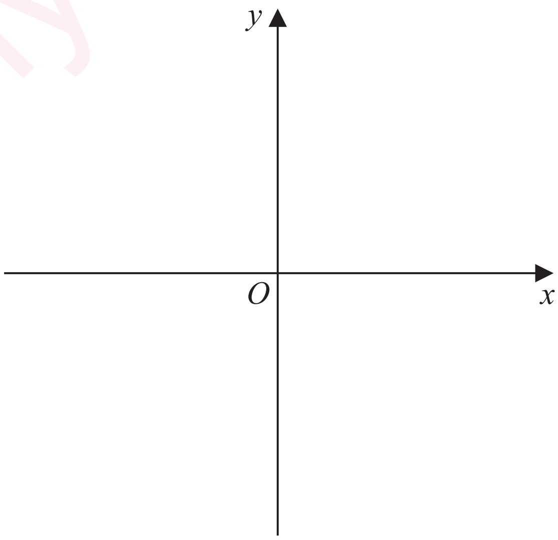
$\therefore$ this is x-coordinate of Q.

Find y-coordinate of Q by substituting $x = 5/2$

$$y = 2x + 7 = 2(5/2) + 7 = 5 + 7 = 12$$

$$\therefore Q\left(5/2, 12\right)$$

Only use this copy of Diagram 1 if you need to redraw your graph.



Copy of Diagram 1

(Total for Question 7 is 10 marks)

8.

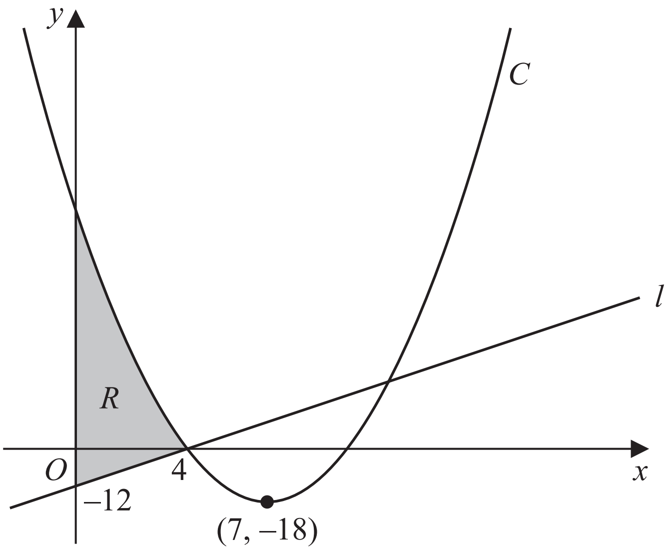


Figure 2

Figure 2 shows a sketch of the straight line l and the curve C.

Given that l cuts the y-axis at –12 and cuts the x-axis at 4, as shown in Figure 2,

(a) find an equation for l, writing your answer in the form $y = mx + c$, where m and c are constants to be found.

(2)

Given that C

- has equation $y = \mathrm{f}(x)$ where $\mathrm{f}(x)$ is a quadratic expression
- has a minimum point at (7, –18)
- cuts the x-axis at 4 and at k, where k is a constant

(b) deduce the value of k,

(1)

(c) find $\mathrm{f}(x)$.

(3)

The region R is shown shaded in Figure 2.

(d) Use inequalities to define R.

(2)

a) Coordinates of l are $(0,-12)$ & $(4,0)$

① find gradient using gradient formula $m = \dfrac{y_1 - y_2}{x_1 - x_2}$

$m = \dfrac{-12 - 0}{0 - 4} = \dfrac{-12}{-4} = 3$

Question 8 continued

② Find equation of line using line passing through (a, b) & gradient M

equation : $(y-b) = M(x-a)$

$a = 4$
$b = 0$
$M = 3$

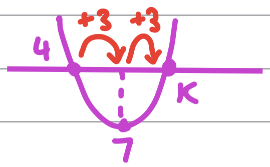
$(y-0) = 3(x-4)$

③ Write in form $y = mx + c$

$y = 3(x-4)$
$y = 3x - 12$

$\therefore y = 3x - 12$

b)



K is $7+3$

$\therefore K = 10$

c) • $f(x)$ is quadratic so $a(x+b)^2 + c$ form.

• minimum $(7, -18)$ so: $y = a(x+b)^2 + c$

inverse is x ; y

Explanation : when $y = c$

$c = (x+b)^2 + c$
$0 = (x+b)^2$
$\therefore x = -b$

when $x = b$ $\quad y = (b+b)^2 + c$
$\therefore y = c$

So : $y = a(x-7)^2 - 18$

k=10 from part (b)

• Cuts x-axis at 4 & K $\therefore$ passes through $(4, 0)$ & $(10, 0)$

for $(4,0)$: $0 = a(4-7)^2 - 18 \rightarrow 0 = a(-3)^2 - 18 \rightarrow 0 = 9a - 18$

for $(10,0)$. $0 = a(10-7)^2 - 18 \rightarrow 0 = a(3)^2 - 18 \rightarrow 0 = 9a - 18$

$0 = 9a - 18$

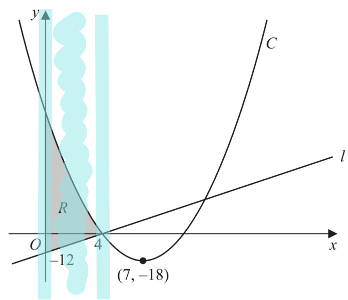
$9a = 18 \xrightarrow{\div 9} a = 2$

$\therefore y = 2(x-7)^2 - 18$

Question 8 continued

d) 3 inequalities to be found

① First inequality



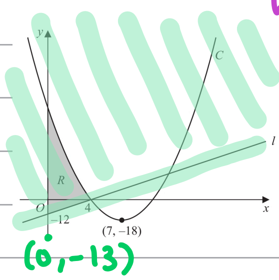
On x-axis, valid region is between $x=0$ & $x=4$

$\therefore \; 0 \leq x \leq 4$

* ↳ $\leq$ ∵ line is solid & not dashed (mark scheme allows $<$, but $\leq$ is more accurate)

[Solid dashed]

② Second inequality



$y = 3x - 12$.

To find inequality, select point OUTSIDE the valid region & make the inequality FALSE.

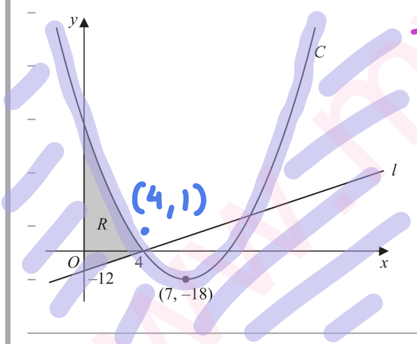
Point chosen is: $(0,-13)$

$$y = 3x - 12$$
$$-13 = 3(0) - 12$$
$$-13 = -12$$

Make FALSE inequality: $-13 \geq -12$

$\therefore \; y \geq 3x - 12$

③ Third inequality



$y = 2(x-7)^2 - 18$

To find inequality, select point OUTSIDE the valid region & make the inequality FALSE.

Point chosen is: $(4,1)$

$$y = 2(x-7)^2 - 18$$
$$1 = 2((4)-7)^2 - 18$$
$$1 = 0$$

Make FALSE inequality: $1 \leq 0$

$\therefore \; y \leq 2(x-7)^2 - 18$

Combine the two inequalities

$$\therefore \; 3x - 12 \leq y \leq 2(x-7)^2 - 18 \quad \& \quad 0 \leq x \leq 4$$

Question 8 continued

(Total for Question 8 is 8 marks)

9.

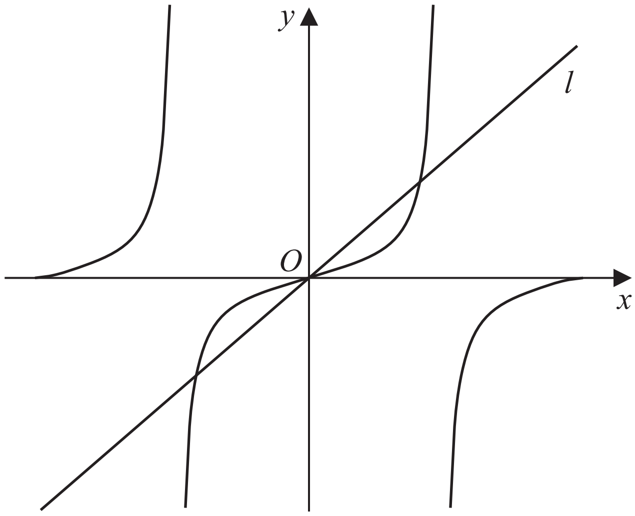


Figure 3

Figure 3 shows a sketch of

- the curve with equation $y = \tan x$
- the straight line l with equation $y = \pi x$

in the interval $-\pi < x < \pi$ ← unit is radians

(a) State the period of $\tan x$

↳ length of interval between repeating waves. **(1)**

(b) Write down the number of roots of the equation

(i) $\tan x = (\pi + 2)x$ in the interval $-\pi < x < \pi$

(1)

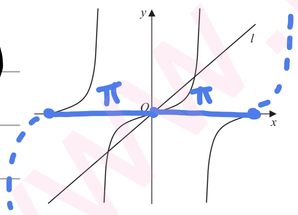
(ii) $\tan x = \pi x$ in the interval $-2\pi < x < 2\pi$

(1)

(iii) $\tan x = \pi x$ in the interval $-100\pi < x < 100\pi$

(1)

a)



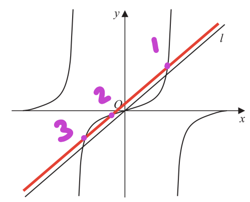
distance for repeating shape

∴ Period is π

b) number of roots in number of intersections in interval.

i) $\tan x = (\pi + 2)x$

if $f(x) = \pi x$, $(\pi+2)x$ is $f(x+2)$ which is translation 2 units to the left.



∴ 3 roots

P 7 2 0 6 6 A 0 2 4 3 2

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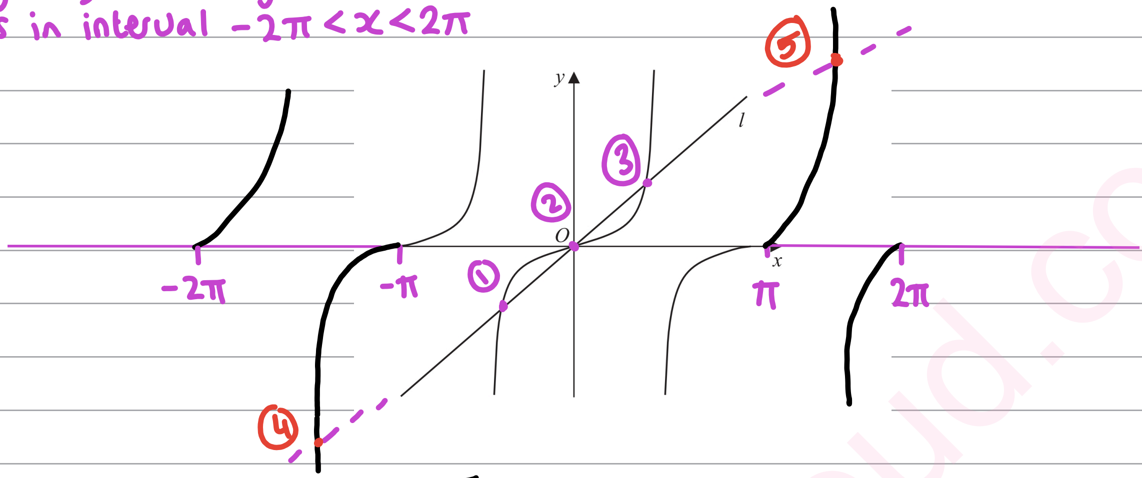
Question 9 continued

ii) $\tan x = \pi x$ in interval $-2\pi < x < 2\pi$

in interval $-\pi < x < \pi$, there are 3 intersections so 3 roots (as shown in graph). in every π interval (besides origin) there is 1 intersection $\therefore$ 2 more roots in interval $-2\pi < x < 2\pi$



$\therefore$ 5 roots

iii) $\tan x = \pi x$ in interval $-100\pi < x < 100\pi$

excluding origin $(0,0)$, in interval $-2\pi < x < 2\pi$ there are 4 roots.

$2\pi \xrightarrow{\times 50} 100\pi$

$4 \xrightarrow{\times 50} 200$

including origin root : $200 + 1 = 201$

$\therefore$ 201 roots

(Total for Question 9 is 4 marks)

10.

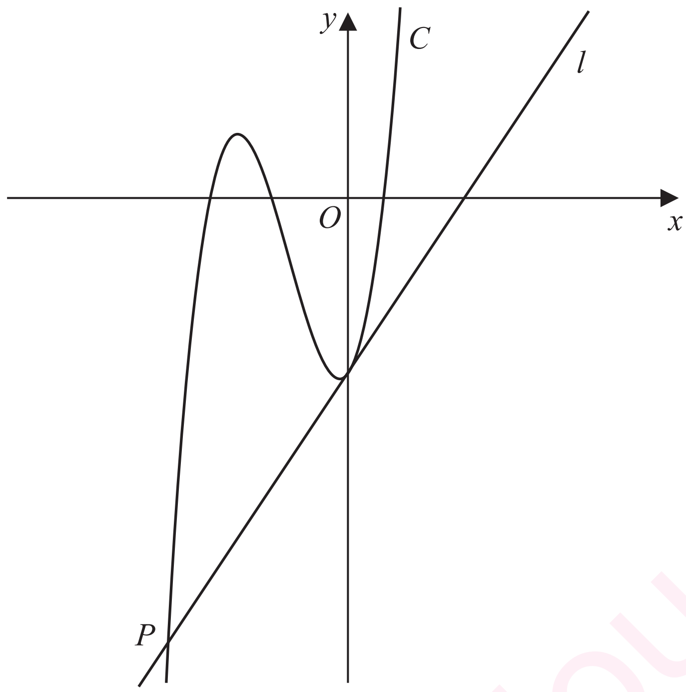


Figure 4

Figure 4 shows a sketch of part of the curve C with equation $y = f(x)$, where

$$f(x) = (3x + 20)(x + 6)(2x - 3)$$

(a) Use the given information to state the values of x for which

$$f(x) > 0$$

(2)

(b) Expand $(3x + 20)(x + 6)(2x - 3)$, writing your answer as a polynomial in simplest form.

(3)

$y = mx + c$

The straight line l is the tangent to C at the point where C cuts the y-axis.

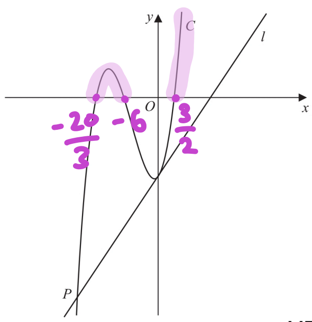
Given that l cuts C at the point P, as shown in Figure 4,

(c) find, using algebra, the x coordinate of P

(Solutions based on calculator technology are not acceptable.)

(5)

a) $f(x) > 0$ means $y > 0$



Points where curve C crosses the x-axis (y=0).

$f(x) = (3x+20)(x+6)(2x-3) = 0$

• $3x+20=0$ $\therefore x = -\frac{20}{3}$

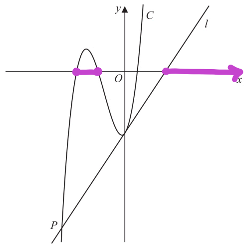
• $x+6=0$ $\therefore x = -6$

• $2x-3=0$ $\therefore x = \frac{3}{2}$

Question 10 continued

Values of x for which $y>0$

NOT $\leq$ $\because y$ is not also equal to 0 (y is NOT $y \geq 0$)



$\therefore$ $-\frac{20}{3} < x < -6$ & $\frac{3}{2} < x$

b) Expanding brackets.

$(3x+20)(x+6)(2x-3) = (3x^2+18x+20x+120)(2x-3)$

$= (3x^2+38x+120)(2x-3) = 6x^3-9x^2+76x^2-114x+240x-360$

$= 6x^3+67x^2+126x-360$

$\therefore (3x+20)(x+6)(2x-3) = 6x^3+67x^2+126x-360$

c) Find equation of l

① l meets (tangent to) curve C when $x=0$

when $x=0$ for curve C: $f(0) = (3(0)+20)(0+6)(2(0)-3)$

$= (20)(6)(-3)$

$= -360$

$\therefore$ l is tangent to C at $(0,-360)$

② find gradient of l.

tangent means gradient of tangent is same as gradient of equation

to find gradient of tangent, substitute x-value of $(0,-360)$ into dy/dx (the gradient function)

$\because x^0 = 1$

$y = 6x^3+67x^2+126x^1-360x^0$

$dy/dx = 3(6x^{3-1}) + 2(67x^{2-1}) + 1(126x^{1-1}) + 0(360x^{0-1})$

$= 18x^2+134x+126$

$dy/dx\big|_{x=0} = 18(0)^2+134(0)+126 = 126$

$\therefore$ gradient is 126

Question 10 continued

③ form equation for L using line passing through (a, b) and gradient M.

equation: $(y-b) = M(x-a)$

$a = 0$
$b = -360$
$M = 126$

$$(y-(-360)) = 126(x-0)$$

$$y + 360 = 126x$$

$$\therefore \quad y = 126x - 360$$

To find point P:

① equate line L & curve C then simplify.

$$126x - 360 = 6x^3 + 67x^2 + 126x - 360$$

$-126x$, $-126x$

$$-360 = 6x^3 + 67x^2 - 360$$

$+360$, $+360$

$$0 = 6x^3 + 67x^2$$

$$6x^3 + 67x^2 = 0$$

② Solve for x.

factorise: $x^2(6x+67) = 0$

Solve: • $x^2 = 0$ • $6x + 67 = 0$

$\therefore x = 0$ $\quad\quad \therefore x = -\frac{67}{6}$

↳ this is x-coordinate when L & C cut y-axis

$\therefore$ x-coordinate of P is $-\frac{67}{6}$

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Question 10 continued

(Total for Question 10 is 10 marks)

11. A curve C has equation $y = f(x)$, $\quad x > 0$

Given that

- $f''(x) = 4x + \dfrac{1}{\sqrt{x}}$
- the point P has x coordinate 4 and lies on C
- the tangent to C at P has equation $y = 3x + 4$

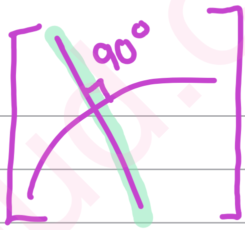
(a) find an equation of the normal to C at P

(2)

(b) find $f(x)$, writing your answer in simplest form.

(6)

a) normal is Perpendicular to Curve



∴ we find gradient of normal (m_n)
using Perpendicular gradient rule $m_{normal} \times m_{curve} = -1$

① Find gradient of normal at P

tangent to C at P has gradient 3 ∵ $y = 3x + 4$
in form $y = mx + c$, m is the gradient.

∴ gradient of C at P is also 3 ∵ tangent means gradient of tangent is $\underline{\underline{\text{same}}}$ as gradient of Curve

Using perpendicular gradient rule: $m_n \times 3 = -1$

$\therefore m_n = -1/3$

② We know x-coordinate of P is 4. Find y-coordinate by substituting $x = 4$ into equation

$y = 3x + 4$

$y = 3(4) + 4 = 12 + 4 = 16$

$\therefore P(4, 16)$

Question 11 continued

③ find equation of tangent using line passing through (a, b) & gradient m

equation: $(y - b) = m(x - a)$

$a = 4$
$b = 16$
$m = -1/3$

$(y - 16) = -1/3(x - 4)$

$\therefore y - 16 = -\frac{1}{3}(x - 4)$ → form not specified so this is a valid answer

b) $f(x) \underset{\text{integrate}}{\overset{\text{differentiate}}{\rightleftarrows}} f'(x) \underset{\text{integrate}}{\overset{\text{differentiate}}{\rightleftarrows}} f''(x)$

$\boxed{1}$ $f''(x) \longrightarrow f'(x)$

$f''(x) = 4x + \frac{1}{\sqrt{x}}$

① write in easier form for integration.

$f''(x) = 4x + \frac{1}{\sqrt{x}} = 4x + \frac{1}{x^{1/2}} = 4x + x^{-1/2}$

① indicies rule: $\sqrt[c]{a^b} = a^{\frac{b}{c}}$ ② indicies rule: $\frac{a}{x^b} = ax^{-b}$

② Integrate

$f'(x) = \int f''(x)\,dx = \int 4x^1 + 1x^{-1/2}\,dx = \left[\left(\frac{4}{1+1}x^{1+1}\right) + \left(\frac{1}{-1/2+1}x^{-1/2+1}\right)\right] = 2x^2 + 2x^{1/2} + c$

③ find $+c$. We know from part (a) that when $x = 4$, gradient of C is 3. $f'(x)$ is the gradient function $\therefore f(4) = 3$

$f(4) = 2(4)^2 + 2(4)^{1/2} + c = 3$
$32 + 4 + c = 3 \Rightarrow 36 + c = 3$
$\therefore c = -33$

$\therefore f'(x) = 2x^2 + 2x^{1/2} - 33$

P 7 2 0 6 6 A 0 3 1 3 2

Question 11 continued

$\boxed{2}$ $f'(x) \longrightarrow f(x)$

① Integrate

$\because x^0 = 1$

$$f(x) = \int f'(x)\,dx = \int 2x^2 + 2x^{1/2} - 33x^0\,dx = \left[\left(\frac{2}{2+1}x^{2+1}\right) + \left(\frac{2}{\frac{1}{2}+1}x^{\frac{1}{2}+1}\right) + \left(\frac{-33}{0+1}x^{0+1}\right)\right]$$

$$= \frac{2}{3}x^3 + \frac{4}{3}x^{3/2} - 33x + C$$

② find value of $+C$.

Do so by substituting $P(4,16)$ into $f(x)$ so that $f(4) = 16$

$$f(4) = 2/3(4)^3 + 4/3(4)^{3/2} - 33(4) + C = 16$$

$$128/3 + 32/3 - 132 + C = 16$$

$$-236/3 + C = 16$$

$+236/3$ $+236/3$

$$C = \frac{284}{3}$$

$$\therefore f(x) = \frac{2}{3}x^3 + \frac{4}{3}x^{3/2} - 33x + \frac{284}{3}$$

(Total for Question 11 is 8 marks)

TOTAL FOR PAPER IS 75 MARKS